Entropic Corrections to Coulomb's Law

S. H. Hendi[1,2*] and A. Sheykhi[2,3†]

[1] *Physics Department, College of Sciences, Yasouj University, Yasouj 75914, Iran*

[2] *Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran*

[3] *Department of Physics, Shahid Bahonar University, Kerman 76175, Iran*

Abstract

Two well-known quantum corrections to the area law have been introduced in the literatures, namely, logarithmic and power-law corrections. Logarithmic corrections, arises from loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations, while, power-law correction appears in dealing with the entanglement of quantum fields in and out the horizon. Inspired by Verlinde's argument on the entropic force, and assuming the quantum corrected relation for the entropy, we propose the entropic origin for the Coulomb's law in this note. Also we investigate the Uehling potential as a radiative correction to Coulomb potential in 1-loop order and show that for some value of distance the entropic corrections of the Coulomb's law is compatible with the vacuum-polarization correction in QED. So, we derive modified Coulomb's law as well as the entropy corrected Poisson's equation which governing the evolution of the scalar potential ϕ. Our study further supports the unification of gravity and electromagnetic interactions based on the holographic principle.

* E-mail: hendi@mail.yu.ac.ir

† E-mail: sheykhi@mail.uk.ac.ir

I. INTRODUCTION

The profound connection between gravity and thermodynamics has a long history since the discovery of black holes thermodynamics in 1970's [1–3]. It was discovered that black holes can emit Hawking radiation with a temperature proportional to its surface gravity at the black hole horizon and black hole has an entropy proportional to its horizon area [2]. The Hawking temperature and horizon entropy together with the black hole mass obey the first law of black hole thermodynamics [3]. The studies on the connection between gravity and thermodynamics has been continued until in 1995 Jacobson showed that the Einstein field equation is just an equation of state for spacetime and in particular it can be derived from the the first law of thermodynamics together with relation between the horizon area and entropy [4]. Following Jacobson, however, several recent investigations have shown that there is indeed a deeper connection between gravitational dynamics and horizon thermodynamics. It has been shown that the gravitational field equations in a wide variety of theories, when evaluated on a horizon, reduce to the first law of thermodynamics and vice versa. This result, first pointed out in [5], has now been demonstrated in various theory including f(R) gravity [6], cosmological setups [7–12], and in braneworld scenarios [13, 14]. For a recent review on the thermodynamical aspects of gravity and complete list of references see [15]. Although Jacobson's derivation is logically clear and theoretically sound, the statistical mechanical origin of the thermodynamic nature of gravity remains obscure.

A constructive new idea on the relation between gravity and thermodynamics was recently proposed by Verlinde [16] who claimed that gravity is not a fundamental interaction and can be interpreted as an entropic force arising from the change of information when a material body moves away from the holographic screen. Verlinde postulated that when a test particle approaches a holographic screen from a distance $\triangle x$, the magnitude of the entropic force on this body has the form

$$F\triangle x = T\triangle S, \tag{1}$$

where T and $\triangle S$ are the temperature and the entropy change on the screen, respectively (see Fig. 1).

Focusing on the physical explanation of interesting proposal of Verlinde, it has been shown that his idea is problematic [17, 18]. In other word, although Verlinde's derivation is right, mathematically, it does not prove that gravity is an entropic force, physically. We should

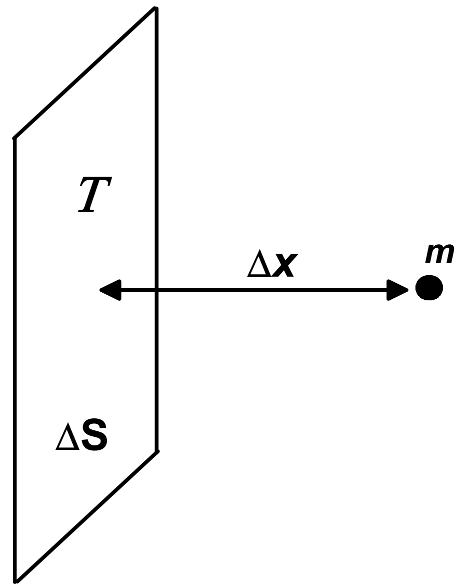

FIG. 1: Test particle with mass m approaches a holographic screen with the temperature T and the entropy change $\triangle S$.

note that it has been presented a general objection to viewing gravity as an entropic force [18] and it has been proved that Verlinde's idea is supported by a mathematical argument based on a discrete group theory [19]. In addition, considering a modified entropic force with the covariant entropy bound, one may obtain the Newtonian force law [20]. Also, following the controversial hypothesis in Ref. [21], it has been shown that gravity is an entropic force.

Verlinde's derivation of laws of gravitation opens a new window to understand gravity from the first principles. The entropic interpretation of gravity has been used to extract Friedmann equations at the apparent horizon of the Friedmann-Robertson-Walker universe [22], modified Friedmann equations [23], modified Newton's law [24], the Newtonian gravity in loop quantum gravity [25], the holographic dark energy [26], thermodynamics of black holes [27] and the extension to Coulomb force [28]. Other studies on the entropic force have been carried out in [29].

In addition, the derivation of Newton's law of gravity, in Verlinde's approach, depends on the entropy-area relationship $S = A/4\ell_p^2$ of black holes in Einstein's gravity, where $A = 4\pi R^2$ represents the area of the horizon and $\ell_p^2 = G\hbar/c^3$ is the Planck length. However, this definition can be modified from the inclusion of quantum effects. Two well-known quantum corrections to the area law have been introduced in the literatures, namely, logarithmic and

power-law corrections. Logarithmic corrections, arises from loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations [30, 31],

$$S = \frac{A}{4\ell_p^2} - \beta \ln \frac{A}{4\ell_p^2} + \gamma \frac{\ell_p^2}{A} + \text{const}, \tag{2}$$

where β and γ are dimensionless constants of order unity. The exact values of these constants are not yet determined and still an open issue in quantum gravity.

Power-law correction appears in dealing with the entanglement of quantum fields in and out the horizon. The entanglement entropy of the ground state obeys the Bekenstein-Hawking area law. However, a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states [32]. In other words, the excited state contributes to the power-law correction, and more excitations produce more deviation from the area law [33]. The power-law corrected entropy is written as [32, 34]

$$S = \frac{A}{4\ell_p^2}\left[1 - K_\alpha A^{1-\alpha/2}\right] \tag{3}$$

where α is a dimensionless constant whose value ranges as $2 < \alpha < 4$ [32], and

$$K_\alpha = \frac{\alpha (4\pi)^{\alpha/2-1}}{(4-\alpha) r_c^{2-\alpha}} \tag{4}$$

where r_c is the crossover scale. The second term in Eq. (3) can be regarded as a power-law correction to the area law, resulting from entanglement, when the wave-function of the field is chosen to be a superposition of ground state and exited state [32]. Taking the corrected entropy-area relation into account, the corrections to the Newton's law of gravitation as well as the modified Friedman equations were derived [23].

In this paper, we would like to extend the study to the electromagnetic interaction. We will derive the general quantum corrections to the Coulomb's law, Poisson's equation and the general form of the modified Newton-Coulomb's law by assuming the entropic origin for the electromagnetic interaction.

II. ENTROPIC CORRECTIONS TO COULOMB'S LAW

In order to derive the corrections to the Coulomb's law of electromagnetic, we consider the modified entropy-area relationship in the following form

$$S = \frac{A}{4\ell_p^2} + s(A), \tag{5}$$

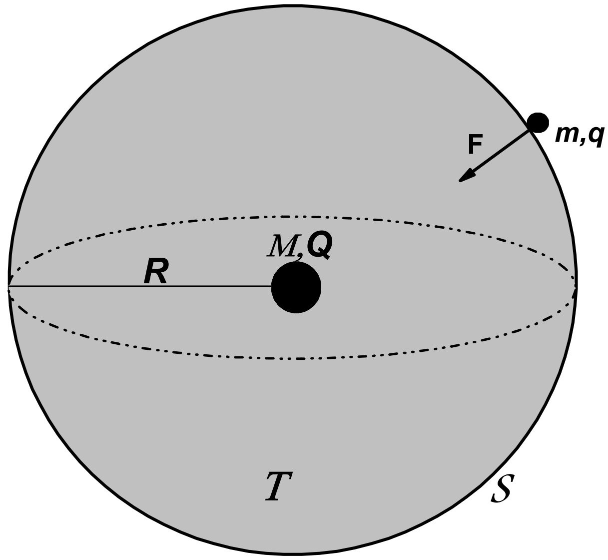

FIG. 2: A test charged particle with mass m and charge q and the other considered as the source with respective charge Q and mass M surrounded by the spherically symmetric screen $\mathcal{S}$.

where $s(A)$ represents the general quantum correction terms in the entropy expression. We assume there are two charged particles, one a test charged particle with mass m and charge q and the other considered as the source with respective charge Q and mass M located at the center (see Fig. 2 for more details). Centered around the source mass M with charge Q, is a spherically symmetric surface $\mathcal{S}$ which will be defined with certain properties that will be specified explicitly later. To derive the entropic law, the surface $\mathcal{S}$ is between the test mass and the source mass, but the test mass is assumed to be very close to the surface as compared to its reduced Compton wavelength $\lambda_m = \frac{\hbar}{mc}$. When a test mass m is a distance $\triangle x = \eta \lambda_m$ away from the surface $\mathcal{S}$, the entropy of the surface changes by one fundamental unit $\triangle S$ fixed by the discrete spectrum of the area of the surface via the relation

$$\triangle S = \frac{\partial S}{\partial A} \triangle A = \left(\frac{1}{4\ell_p^2} + \frac{\partial s(A)}{\partial A} \right) \triangle A. \tag{6}$$

We find out that in order to interpret the entropic origin for the electromagnetic force, we should leave away the relativistic rest mass energy $E = Mc^2$, and instead, in a similar manner, we propose the relativistic rest electromagnetic energy of the source Q as

$$E = \Gamma Q c^2, \tag{7}$$

where $\Gamma = \chi q/m$, and χ is a constant with known dimension ($[\chi] = \frac{[k]}{[G]}$, where k and G are Coulomb and Newtonian constants, respectively). Although the physical interpretation of

assumption (7) is not clear well yet for us, however as we will see it leads to the reasonable results. It is notable to mention that the charge/mass ratio (q/m) is a physical quantity that is widely used in the electrodynamics of charged particles. When a charged particle follows a circular which is caused by the magnetic field, the magnetic force is acting like a centripetal force. It is easy to find that charge/mass ratio $(q/m = V/Br)$ is a constant in which we equal it to Γ/χ.

Considering the relativistic rest mass energy relation with the motivation of analogy between mass in gravity and charge in electromagnetic interactions, one may consider $E_{EM} = \mathcal{M}_{EM}c^2$, in which E_{EM} is the electromagnetic energy and $\mathcal{M}_{EM} = \Gamma Q$ is its corresponding mass which we call it as the electromagnetic mass. It is notable that there are other concepts of mass in special relativity, such as longitudinal mass and transverse mass.

We should mention that we are working in the geometrized unit of charge, in which the Coulomb's law takes almost the same form as the Newton's law except for the difference in signature. On the surface $\mathcal{S}$, there live a set of "bytes" of information that scale proportional to the area of the surface so that

$$A = \xi N, \tag{8}$$

where N represents the number of bytes and ξ is a fundamental constant which should be determined later. Assuming the temperature on the surface is T, and then according to the equipartition law of energy [35], the total energy on the surface is

$$E = \frac{1}{2} N k_B T. \tag{9}$$

Finally, we assume that the electric force on the charge particle q follows from the generic form of the entropic force governed by the thermodynamic equation of state

$$F = T\frac{\triangle S}{\triangle x}, \tag{10}$$

where $\triangle S$ is one fundamental unit of entropy when $|\triangle x| = \eta\lambda_m$, and the entropy gradient points radially from the outside of the surface to inside. Note that N is the number of bytes and thus $\triangle N = 1$; hence from (8) we find $\triangle A = \xi$. Now, we are in a position to derive the entropy-corrected Coulomb's law. Combining Eqs. (6)- (10), we reach

$$\begin{aligned} F &= \frac{2\Gamma Q c^2}{N k_B}\frac{\Delta A}{\Delta x}\left(\frac{\partial S}{\partial A}\right) \\ &= \frac{2\Gamma Q \xi m c^3}{N k_B \eta \hbar}\left(\frac{\partial S}{\partial A}\right) \end{aligned}$$

$$= \frac{Qq}{R^2}\left(\frac{\chi\xi^2 c^3}{8\pi k_B\eta\hbar\ell_p^2}\right)\left[1+4\ell_p^2\frac{\partial s}{\partial A}\right]_{A=4\pi R^2}, \tag{11}$$

This is nothing but the Coulomb's law of electromagnetic to the first order provided we define $\xi^2 = 8\pi k_B\eta\ell_p^4$ and $\chi = 1/(4\pi\varepsilon_0 G) = \hbar/(4\pi\varepsilon_0\ell_p^2 c^3)$. Thus we write the general quantum corrected Coulomb's law as

$$F_{\rm em} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}\left[1+4\ell_p^2\frac{\partial s}{\partial A}\right]_{A=4\pi R^2}. \tag{12}$$

In order to specify the correction terms explicitly, we use the two well-known kinds of entropy corrections. It is easy to show that

$$F_{\rm em1} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}\left[1-\frac{\beta}{\pi}\frac{\ell_p^2}{R^2}-\frac{\gamma}{4\pi^2}\frac{\ell_p^4}{R^4}\right], \tag{13}$$

$$F_{\rm em2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}\left[1-\frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{14}$$

where $F_{\rm em1}$ and $F_{\rm em2}$ are, respectively, the logarithmic and power-law corrected Coulomb's law. Thus, with the corrections in the entropy expression, we see that the Coulomb's law will be modified accordingly. Since the correction terms in Eqs. (13) and (14) can be comparable to the first term only when R is very small (i.e. $R << l_p$ and $R << r_c$ for Eqs. (13) and (14), respectively), the corrections make sense only at the very small distances (note that $\alpha > 2$). For large distances (i.e. $R >> l_p$ for (13) and $R >> r_c$ for (14)), the entropy-corrected Coulomb's law reduces to the usual Coulomb's law of electromagnetic.

III. UEHLING CORRECTION TO COULOMB'S LAW

In order to compare the entropic correction with QED correction of Coulomb's law, we introduce the the so called Uehling potential [36] as a radiative correction to Coulomb potential in 1-loop order (the vacuum-polarization correction for an electron in a nuclear Coulomb field). Using the Born approximation, the relation between the scattering amplitude M and the potential is given by

$$< p'\,|iM|\,p >= -i2\pi V(\mathbf{q})\delta(E_{p'}-E_p), \tag{15}$$

where p (p') and E_p ($E_{p'}$) are the momenta and energy of the incoming (outgoing) particles, respectively, and $\mathbf{q} = \mathbf{p'} - \mathbf{p}$. For ordinary QED, the amplitude of a particle-antiparticle

scattering is given by [37]

$$iM \sim -\frac{ie^2}{|\mathbf{p}' - \mathbf{p}|^2}. \tag{16}$$

Comparing (16) with (15), one can show that the attractive classical Coulomb potential $V(\mathbf{q})$ is given by

$$V(\mathbf{q}) = -\frac{e^2}{|\mathbf{q}|^2}, \tag{17}$$

where $|\mathbf{q}| = |\mathbf{p} - \mathbf{p}'|$. Using a Fourier transformation into the coordinate space, one can find

$$V(\mathbf{x}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{q}) e^{i\mathbf{q}.\mathbf{x}} = -\frac{\alpha'}{\mathbf{R}}, \tag{18}$$

where $\mathbf{R} = |\mathbf{x}|$ and α' is the fine structure constant. Furthermore, to include the quantum correction into the result, the modified Coulomb potential can be calculated from

$$V(\mathbf{x}) = -e^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}.\mathbf{x}}}{\mathbf{q}^2[1 - \Pi(\mathbf{q}^2)]}, \tag{19}$$

where $\Pi(\mathbf{q})$ in the ordinary QED is defined by the vacuum polarization tensor

$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2), \tag{20}$$

and is given by

$$\Pi(q^2) = -\frac{2\alpha'}{\pi} \int_0^1 x(1-x) \log\left(\frac{m^2}{m^2 - x(1-x)q^2}\right) dx. \tag{21}$$

Choosing $q_0 = 0$ and inserting this relation into (19), after some straightforward calculation [37], one can obtain the so called Uehling potential

$$V(R) = -\frac{\alpha'}{R}\left(1 + \frac{\alpha'}{4\sqrt{\pi}} \frac{e^{-2mR}}{(mR)^{3/2}} + ...\right), \tag{22}$$

and after differentiation we can obtain the corresponding Uehling force

$$F_{Ueh} = \frac{\alpha'}{R^2}\left(1 - \frac{\alpha'}{8\sqrt{\pi}} \frac{e^{-2mR}}{(mR)^{3/2}}(4mR + 5) + ...\right). \tag{23}$$

In order to compare the results of the entropic and the Uehling corrections, we can plot the corresponding forces for different values of distance R. We draw three logarithmic figures for different scale. Figure 3, which is drawn for very small scale ($0 < R < 10^{-5}$), shows that for small value of R, F_{em2} is compatible with F_{Ueh} and for a special value of R they are equal, and also F_{em1} is near to the Coloumb force, F_{col}. When we investigate the figure

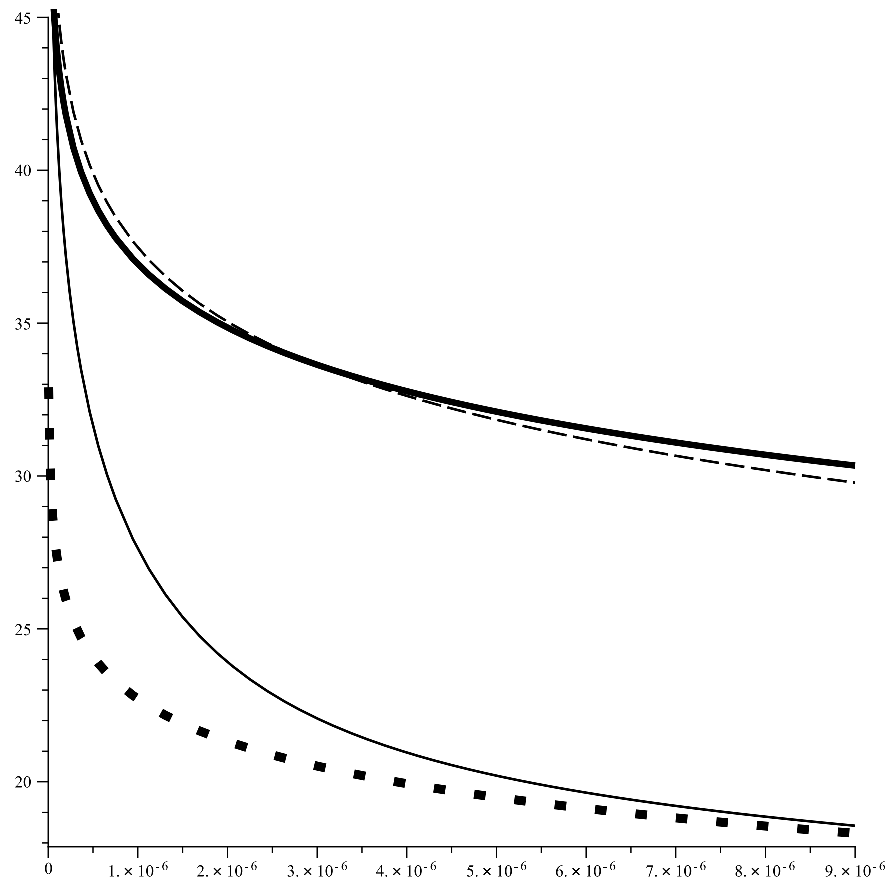

FIG. 3: $\ln F_{em1}$ (solid line), $\ln F_{em2}$ (bold line), $\ln F_{Coulomb}$ (dotted line) and $\ln F_{Ueh}$ (dashed line) versus R ($0 < R < 10^{-5}$) for $\hbar = c = 1$, $\beta = -1$, $\gamma = -1$, $r_c = -1$, $\alpha = 3$, $\alpha' = 1/137$ and $m = 1$.

4, which is drawn for medium scale ($0 < R < 0.1$), we find that in this scale, F_{em2} is far from others. One can find that in figure 5, which is plotted for large scale ($0 < R < 3$), F_{em1}, F_{Ueh} and F_{col} are overlapped to each other and F_{em2} is separated. These figures show that for small values of distance F_{em2} is more compatible with Uehling force, but for large values of R, F_{em1} is more near to F_{Ueh}. As a result it is interesting to study the entropic force arising from the change of information.

IV. GENERALIZED EQUIPARTITION RULE AND NEWTON-COULOMB'S LAW

In this section we would like to generalize our discussion in the previous section to the case where electromagnetic force as well as the gravitational force are considered. We will

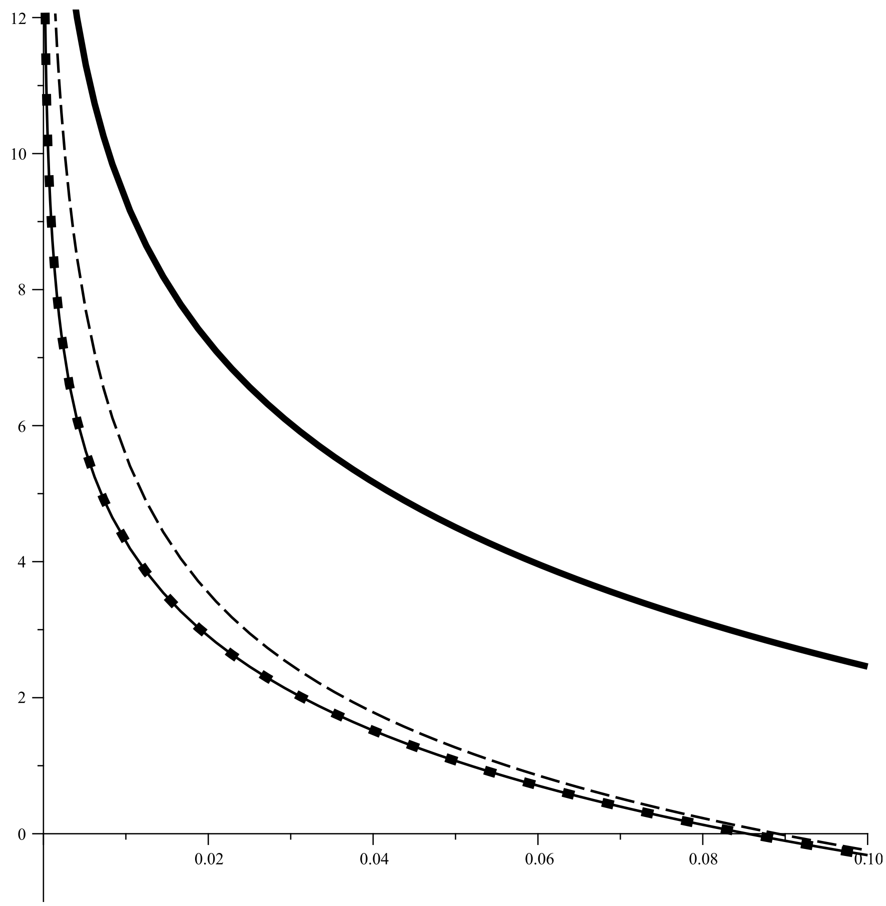

FIG. 4: $\ln F_{em1}$ (solid line), $\ln F_{em2}$ (bold line), $\ln F_{Coulomb}$ (dotted line) and $\ln F_{Ueh}$ (dashed line) versus R $(0 < R < 0.1)$ for $\hbar = c = 1$, $\beta = -1$, $\gamma = -1$, $r_c = -1$, $\alpha = 3$, $\alpha' = 1/137$ and $m = 1$.

study two approaches in dealing with the problem.

A. First Approach

In the first approach we identify the total relativistic rest energy as

$$E = Mc^2 + \Gamma Q c^2, \tag{24}$$

and thus the equipartition rule (7) will be replaced with

$$Mc^2 + \Gamma Q c^2 = \frac{1}{2} N k_B T, \tag{25}$$

Inserting Eqs. (24)- (25) in Eq. (10) after using Eqs. (6) and (8), we find

$$F_{\text{g,em}} = \frac{2\,(M + \Gamma Q)\, c^2}{N k_B} \frac{\Delta A}{\Delta x} \left(\frac{\partial S}{\partial A}\right)$$

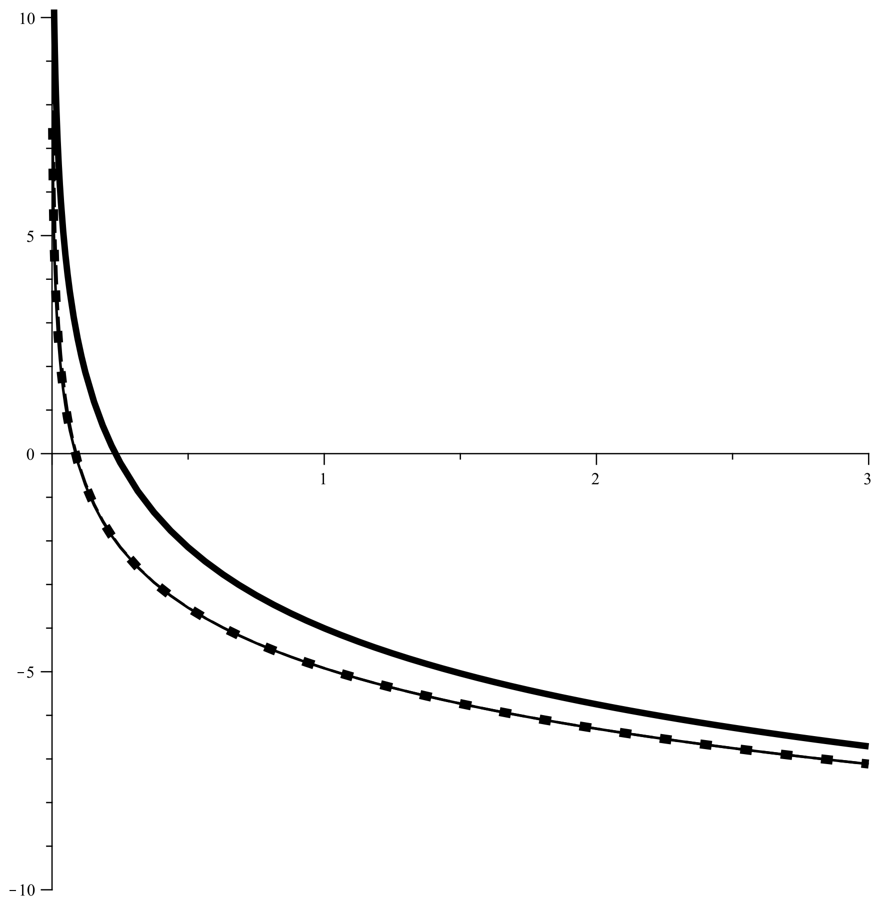

FIG. 5: $\ln F_{em1}$ (solid line), $\ln F_{em2}$ (bold line), $\ln F_{Coulomb}$ (dotted line) and $\ln F_{Ueh}$ (dashed line) versus R ($0 < R < 3$) for $\hbar = c = 1$, $\beta = -1$, $\gamma = -1$, $r_c = -1$, $\alpha = 3$, $\alpha' = 1/137$ and $m = 1$.

$$
\begin{aligned}
&= \frac{2\,(M+\Gamma Q)\,\xi m c^3}{N k_B \eta \hbar}\left(\frac{\partial S}{\partial A}\right) \\
&= \frac{(mM+\chi q Q)}{R^2}\left(\frac{\xi^2 c^3}{8\pi k_B \eta \hbar \ell_p^2}\right)\left[1 + 4\ell_p^2 \frac{\partial s}{\partial A}\right]_{A=4\pi R^2}
\end{aligned}
\tag{26}
$$

Again if we define $\xi^2 = 8\pi k_B \eta \ell_p^4$ and $\chi = k/G = \hbar/(4\pi\varepsilon_0 \ell_p^2 c^3)$, after also using Eqs. (2) and (3), we reach directly the modified Newton-Coulomb's law corresponding to the logarithmic and power-law corrections, respectively,

$$
F_{\rm g,em} = \frac{GmM + kqQ}{R^2}\left[1 - \frac{\beta}{\pi}\frac{\ell_p^2}{R^2} - \frac{\gamma}{4\pi^2}\frac{\ell_p^4}{R^4}\right], \tag{27}
$$

$$
F_{\rm g,em} = \frac{GmM + kqQ}{R^2}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]. \tag{28}
$$

These are the total entropy-corrected forces between a test particle with charge q and mass m in a distance R of a source particle with charge Q and mass M. We see that the cor-

rection terms have the same form for both gravitation and electromagnetic forces. If one of the particle does not have charge, i.e. $q = 0$ or $Q = 0$, then Eq. (27) reduces to the quantum correction Newton's law of gravitation [23]. Again we see that the corrections play a significant role only at the very small distances of R.

B. Second Approach

The second approach is very simple. It is sufficient to add the modified electromagnetic force obtained in Eq. (12) and the modified Newton's law of gravitation derived in [23], where in the general form is

$$F_{\rm g} = G\frac{mM}{R^2}\left[1 + 4\ell_p^2 \frac{\partial s}{\partial A}\right]_{A=4\pi R^2}. \tag{29}$$

Since the emergent directions of gravity and electromagnetic forces coincide, we can obtain

$$\begin{aligned} F_{\rm g,em} &= F_{\rm g} + F_{\rm em} \\ &= \frac{GmM + kqQ}{R^2}\left[1 + 4\ell_p^2 \frac{\partial s}{\partial A}\right]_{A=4\pi R^2}. \end{aligned}$$

Using Eqs. (2) and (3), it is straightforward to recover Eqs. (27) and (28).

V. ENTROPY CORRECTED POISSON'S EQUATION

We can also derive the modified Poisson's equation for the electric potential ϕ, provided we define a new wavelength $\lambda_q = \frac{\delta\hbar}{qc}$ instead of Compton wavelength, $\lambda_m = \frac{\hbar}{mc}$, where $\delta = \sqrt{4\pi\varepsilon_0 G}$. This definition may be understood if one accept a correspondence between the role of mass m in gravitational force and the role of charge q in the electromagnetic force. Consider the differential form of Gauss's law

$$\overrightarrow{\nabla}.\overrightarrow{E} = \frac{\rho}{\varepsilon_0}, \tag{30}$$

and the fact that electrical field has zero curl and equivalently $\overrightarrow{E} = -\overrightarrow{\nabla}\phi$, where ϕ is the electrical potential, it is easy to obtain the familiar Poisson's equation as

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}. \tag{31}$$

In this section, by assuming the modified entropy-area relation (5), we want to obtain the modified Poisson's equation. It was argued in [16] that the holographic screens correspond to the equipotential surfaces, so it is natural to define

$$-\frac{\delta N}{2c^2}\nabla\phi = \frac{\Delta S}{\Delta x}, \tag{32}$$

where $\frac{\Delta S}{\Delta x} = \left(\frac{\partial S}{\partial A}\right)\frac{\Delta A}{\Delta x}$. Substituting $N = \frac{A}{\ell_p^2}$, $\Delta A = \ell_p^2$ and $\Delta x = \frac{\lambda_q}{8\pi}$, where $\lambda_q = \frac{\delta\hbar}{qc}$ in Eq. (32), we can rewrite it in the differential from

$$-\frac{\sqrt{\pi\varepsilon_0 G}}{\ell_p^2 c^2}\nabla\phi dA = \frac{4\pi c\ell_p^2}{\sqrt{\pi\varepsilon_0 G}\hbar}\left(\frac{\partial S}{\partial A}\right)dq. \tag{33}$$

Using the divergence theorem, we find

$$-\frac{\sqrt{\pi\varepsilon_0 G}}{\ell_p^2 c^2}\int\nabla^2\phi dV = \frac{4\pi c\ell_p^2 q}{\sqrt{\pi\varepsilon_0 G}\hbar}\left(\frac{\partial S}{\partial A}\right). \tag{34}$$

Now, we are in a position to extract the modified Poisson's equation

$$\nabla^2\phi = -\frac{4\pi\ell_p^4 c^3}{\pi\varepsilon_0 G\hbar}\left(\frac{\partial S}{\partial A}\right)\frac{dq}{dV}, \tag{35}$$

Using Eq. (6), the above equation can be further rewritten

$$\nabla^2\phi = -\frac{\ell_p^2 c^3}{\varepsilon_0 G\hbar}\rho\left[1 + 4l_p^2\frac{\partial s}{\partial A}\right], \tag{36}$$

where we have defined the charge density $\rho = dq/dV$. Finally, using the fact that $c^3\ell_p^2/\hbar = G$, we can write the modified Poisson's equation in the following manner

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}\left[1 + 4l_p^2\frac{\partial s}{\partial A}\right]_{A=4\pi R^2}, \tag{37}$$

where it reduces to

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}\left[1 - \frac{\beta}{\pi}\frac{\ell_p^2}{R^2} - \frac{\gamma}{4\pi^2}\frac{\ell_p^4}{R^4}\right], \tag{38}$$

and

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{39}$$

for logarithmic and power-law corrections, respectively. In this way, one can derive the quantum correction to Poisson's equation.

VI. CONCLUSIONS

To conclude, taking into account the quantum corrections in area law of the black hole entropy, we derived the modified Coulomb's law of electromagnetic as well as the generalized Newton-Coulomb's law in the presence of correction terms. In addition we investigated the vacuum-polarization correction in QED (Uehling potential) and found that the results of entropic corrections of Coulomb's law is near to the Uehling potential for some distances. This compatibility motivated us to investigate the entropic force in other electromagnetic field equations. We also obtained entropy-corrected Poisson's equation which governing the evolution of the scalar potential ϕ. Our study is the quite one generalization of Verlinde's argument on the gravity force, to the electromagnetic interaction. According to the Verlinde's discussion the gravitational force has a holographic origin. In this work we proposed a similar nature for the electromagnetic interaction. Our motivation is the high apparent similarity between the Newton's law and the Coulomb's law. If gravity and electromagnetic interaction can be extracted from holographic principle, this can be regarded as a form unification of gravity and electromagnetic force. Interestingly enough, we found that the correction terms have similar form for both Newton's law and Coulomb's law. This implies that in the very small distances, these two fundamental forces have the same behavior. This fact further supports the unification of gravity and electromagnetic interactions based on the holographic principle.

Acknowledgements

We thank the referees for constructive comments. This work has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

---

[1] J. D. Bekenstein, Phys. Rev. D 7, 2333 (1973);
S. W. Hawking, Commun Math. Phys. 43, 199 (1975);
S. W. Hawking, Nature 248, 30 (1974).

[2] J. M. Bardeen, B. Carter and S. W. Hawking, Commun. Math. Phys. 31, 161 (1973).

[3] P. C. W. Davies, J. Phys. A: Math. Gen. 8, 609 (1975);

W. G. Unruh, Phys. Rev. D 14, 870 (1976);

L. Susskind, J. Math. Phys. 36, 6377 (1995).

[4] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995).

[5] T. Padmanabhan, Class. Quantum. Grav. **19**, 5387 (2002).

[6] C. Eling, R. Guedens and T. Jacobson, Phys. Rev. Lett. **96**, 121301 (2006).

[7] M. Akbar and R. G. Cai, Phys. Rev. D **75**, 084003 (2007).

[8] R. G. Cai and L. M. Cao, Phys.Rev. D **75**, 064008 (2007).

[9] R. G. Cai and S. P. Kim, JHEP **0502**, 050 (2005).

[10] B. Wang, E. Abdalla and R. K. Su, Phys. Lett. B **503**, 394 (2001);

B. Wang, E. Abdalla and R. K. Su, Mod. Phys. Lett. A **17**, 23 (2002).

[11] R. G. Cai, L. M. Cao and Y. P. Hu, JHEP **0808**, 090 (2008).

[12] S. Nojiri and S. D. Odintsov, Gen. Relativ. Gravit. **38**, 1285 (2006);

A. Sheykhi, Class. Quantum Grav. **27**, 025007 (2010);

A. Sheykhi, Eur. Phys. J. C **69**, 265 (2010).

[13] A. Sheykhi, B. Wang and R. G. Cai, Nucl. Phys. B **779**, 1 (2007);

R. G. Cai and L. M. Cao, Nucl. Phys. B **785**, 135 (2007).

[14] A. Sheykhi, B. Wang and R. G. Cai, Phys. Rev. D **76**, 023515 (2007);

A. Sheykhi, B. Wang, Phys. Lett. B **678**, 434 (2009).

[15] T. Padmanabhan, Rept. Prog. Phys. **73**, 046901 (2010).

[16] E. P. Verlinde, JHEP **1104**, 029 (2011).

[17] S. Hossenfelder, [arXiv:1003.1015];

A. Kobakhidze, Phys. Rev. D 83, 021502 (2011);

B. L. Hu, Int. J. Mod. Phys. D 20, 697 (2011);

A. Kobakhidze, [arXiv:1108.4161].

[18] S. Gao, Entropy, 13, 936 (2011).

[19] H. E. Winkelnkemper, AP Theory V: Thermodynamics in Topological Disguise, Gravity from Holography and Entropic Force as Dynamic Dark Energy. Available online:http://www.math.umd.edu/∼hew/ (accessed on 7 April 2011); Preprint, February 2011

[20] Y. S. Myung, Eur. Phys. J. C 71, 1549 (2011).

[21] M. Chaichian, M. Oksanen and A. Tureanu, [arXiv:1109.2794].

[22] R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D **81**, 061501(R) (2010);

Y. Ling and J. P. Wu, JCAP **1008**, 017 (2010).

[23] A. Sheykhi, Phys. Rev. D **81**, 104011 (2010);

A. Sheykhi and S. H. Hendi, Phys. Rev. D **84**, 044023 (2011).

[24] L. Modesto and A. Randono, [arXiv:1003.1998].

[25] L. Smolin, [arXiv:1001.3668].

[26] M. Li and Y. Wang, Phys. Lett. B **687**, 243 (2010);

D. A. Easson, P. H. Frampton and G. F. Smoot, Phys. Lett. B **696**, 273 (2011);

U. H. Danielsson, [arXiv:1003.0668].

[27] Y. Tian and X. Wu, Phys. Rev. D **81**, 104013 (2010);

[28] T. Wang, Phys. Rev. D **81**, 104045 (2010).

[29] Y. X. Liu, Y. Q. Wang, S. W. Wei, Class. Quantum Grav. **27**, 185002 (2010);

V. V. Kiselev and S. A. Timofeev, Mod. Phys. Lett. A **25**, 2223 (2010);

R. A. Konoplya, Eur. Phys. J. C **69**, 555 (2010);

R. Banerjee and B. R. Majhi. Phys. Rev. D **81**, 124006 (2010);

P. Nicolini, Phys. Rev. D **82**, 044030 (2010);

C. Gao, Phys. Rev. D **81**, 087306 (2010);

Y. S. Myung and Y.W Kim, Phys. Rev. D **81**, 105012 (2010);

H. Wei, Phys. Lett. B **692**, 167 (2010);

D. A. Easson, P. H. Frampton and G. F. Smoot, [arXiv:1003.1528];

S. W. Wei, Y. X. Liu and Y. Q. Wang, *to be published in Commun. Theor. Phys.* [arXiv:1001.5238].

[30] K. A. Meissner, Class. Quantum Grav. **21**, 5245 (2004);

A. Ghosh and P. Mitra, Phys. Rev. D **71**, 027502 (2004);

A. Chatterjee and P. Majumdar, Phys. Rev. Lett. **92**, 141301 (2004).

[31] J. Zhang, Phys. Lett. B **668**, 353 (2008);

R. Banerjee and B. R. Majhi, Phys. Lett. B **662**, 62 (2008);

R. Banerjee and B. R. Majhi, JHEP **0806**, 095 (2008);

S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. A **16**, 3273 (2001).

[32] S. Das, S. Shankaranarayanan and S. Sur, Phys. Rev. D **77**, 064013 (2008).

[33] S. Das, S. Shankaranarayanan and S. Sur, [arXiv:1002.1129];

S. Das, S. Shankaranarayanan and S. Sur, [arXiv:0806.0402].

[34] N. Radicella, D. Pavon, Phys. Lett. B **691**, 121 (2010).

[35] T. Padmanabhan, Class. Quantum Grav. **21**, 4485 (2004);
T. Padmanabhan, Mod. Phys. Lett. A **25**, 1129 (2010);
T. Padmanabhan, Phys. Rev. D **81**, 124040 (2010).

[36] E. A. Uehling, Phys. Rev. **48**, 55 (1935);
E. H. Wichmann and N. H. Kroll, Phys. Rev. **101**, 843 (1956);
A. Bonanno and M. Reuter Phys. Rev. D **62**, 043008 (2000);
W. Dittrich and M. Reuter, *Effective Lagrangians in Quantum Electrodynamics*, Springer-Verlag (1985).

[37] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Reading, USA: Addison-Wesley (1995).